IMO Number Grids Questions

Level: Intermediate Ref No: M05 **Puzz Points: 13**

[Cayley 2004 Q5] Four football teams – the Apes, the Baboons, the Chimps and the Gorillas – play each other once in a season. After some of the matches have been played the table of results, with some entries missing, looks like this:

Team	Played	Won	Lost	Drawn	Goals For	Goals Against
A		0		0	2	3
B				0		1
C	2				4	
G		0		1		5

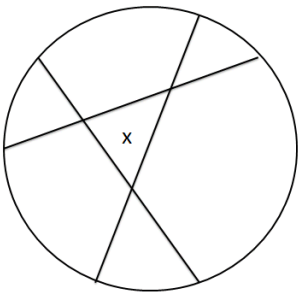
Complete the table, explaining how each entry is worked out, and find the score in each match played so far.

Solution: Each row: (1, 0, 1, 0, 2, 3), (1, 1, 0, 0, 4, 1), (2, 1, 0, 1, 4, 3),(2, 0, 1, 1, 2, 5)

Level: Intermediate Ref No: M30 **Puzz Points: 18**

[Hamilton 2006 Q6] Three chords are drawn in a circle to create seven regions, as shown. The numbers 1 to 7 are to be placed, one in each region, so that, for each chord, the total of the numbers in the circle on one side of the chord is equal to the total of the numbers on the other side.

How many possible values are there for the number *x* in the central region?

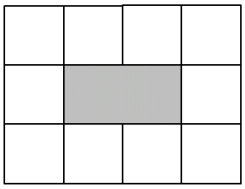


Solution: Three possible values (1, 2 or 4)

Level: Intermediate Ref No: M48 **Puzz Points: 18**

[Hamilton 2007 Q6] The numbers 1 to 10 are to be placed in the unshaded boxes, so that the two rows of four boxes and the two columns of three boxes all have the same total T.

(a) Find a solution when T = 20.
(b) Find the minimum possible value of T.

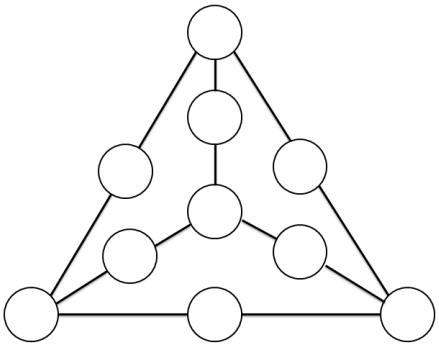


Solution: (a) Any valid arrangement: sum of 4 corners should be 25. (b) 18 (Note that you need a possible solution when $T = 18$)

Level: Intermediate Ref No: M66 **Puzz Points: 18**

[Hamilton 2011 Q6] Sam wishes to place all the numbers from 1 to 10 in the circles, one to each circle, so that each line of three circles has the same total.

Prove that Sam's task is impossible.

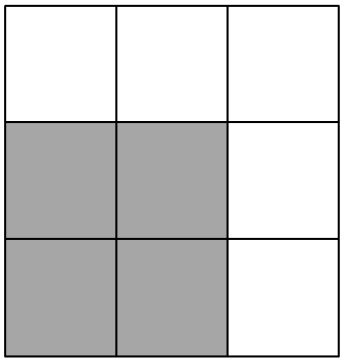


Solution: If three corners a, b, c and centre d, and the total of each line is T, we find $2(a + b + c + d) + 55 = 6T$, which is impossible as the LHS is odd while the RHS is even.

Level: Intermediate Ref No: M72 **Puzz Points: 23**

[Maclaurin 2011 Q6] The numbers 1 to 9 are placed in the cells of a 3×3 square grid, one to each cell. In each of the four 2×2 blocks of adjacent cells, such as the one shaded, the four numbers have the same total *T*.

What is the maximum possible value of *T*?



Solution: First prove $T \leq 24$. Then show 24 works.

Level: Intermediate Ref No: M95 **Puzz Points: 13**

[Cayley 2009 Q6] Teams A, B, C and D competed against each other once. The results table was as follows:

Team	Win	Draw	Loss	Goals For	Goals Against
A	3	0	0	5	1
B	1	1	1	2	2
C	0	2	1	5	6
D	0	1	2	3	6

a) Find (with proof) which team won in each of the six matches.
b) Find (with proof) the scores in each of the six matches.

Solution: (a) A beat B, C and D. B drew with C, B beat D, C drew with D.
(b) A beat B 1-0, A beat C 2-1, A beat D 2-0, B drew with C 1-1, B beat D 1-0, C drew with D 3-3.